

Inequality and the Organization of Knowledge

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Since the seminal work of Katz and Murphy (1992), the study of wage inequality has taken as its starting point a neoclassical CES production function using as inputs capital and low and high skill labor. This approach assumes that the organization of production is fixed and determined by a particular specification of technology, and ignores both the source of the interaction between workers, and the organizational aspects of this interaction. These shortcomings are particularly important in the light of growing empirical evidence that points, first, to the importance of decreases in the cost of processing and communicating information and, second, to the complementarity between organizational change and adjustments in the distribution of wages (e.g. Bresnahan, Brynjolfsson and Hitt (2002)). This paper argues that theories that seek to guide empirical research on these areas must put knowledge and information at the center of the analysis of organizations and link the organizational structure with aggregate variables via equilibrium frameworks.

Garicano and Rossi-Hansberg (2003) presents a model with these characteristics. It determines the organizational structure, as manifested by communication and specialization patterns, and the implied wage structure, that result from different costs of acquiring and communicating information. Here, we present a simple variant of this theory that allows us to focus on one of the main aspects of that framework: the sorting of agents into teams and the wage and organizational structure that accompanies that sorting. We use this simple model to analyze the changes in organization

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and wages that result from a very specific type of technological change: a reduction in the cost of communicating knowledge or information. This model allows us to consider the effect on within-class wage inequality, and the impact of information technology on the creation and form of organizations (e.g. size distribution of hierarchies). However, because knowledge is exogenously given, and agents cannot invest in learning, an important margin of the model in Garicano and Rossi-Hansberg (2003) is fixed, namely, the degree of ‘decentralization’ or the extent to which problems are solved at lower levels. That model allows the simultaneous study of the acquisition of knowledge, spans of control, and matching in equilibrium. Moreover, it goes beyond the current analysis in that it allows for organizations with an unconstrained number of layers, and in that it studies two aspects of the impact of information technology: communication technology (like here) and the technology to acquire knowledge or information (e.g. processing power through cheaper data-base access).

I KNOWLEDGE, COMMUNICATION AND PRODUCTION

We model an economy in which production results from drawing problems and applying production time to them, as in Garicano (2000). Production requires production time and knowledge of the different tasks that must be performed in the course of production. In this version, workers rather than choosing a set of tasks to learn, are endowed with some knowledge. Then they draw a particular task and, possibly, solve it, in which case their production time is useful.

We consider a population that is described by a given distribution of knowledge, $G(z)$, with density function $g(z)$ where $z \in [0, \bar{z}]$ and \bar{z} denotes the maximum amount of knowledge in the population. Agents draw one task per period; the value of learning a task is given by the frequency with which it appears. This frequency is described by a probability density $f(z)$, with associated cumulative distribution function $F(z)$. We assume that more knowledgeable workers always know the tasks of those less knowledgeable ones, so we say that a worker has knowledge z_i when she can perform all tasks in $(0, z_i)$.

Suppose that agents can form teams, so that whenever they do not know how to

solve a problem they can ask other agents. This allows some agents to specialize on handling some problems and not others. We make two assumptions about the communication of knowledge. First, we assume that agents spend a fraction h of their time *communicating* their knowledge about each question posed to them, irrespectively of whether they know the answer or not. Solving a problem does not take time in our setup, time is only useful as an input in production and to communicate knowledge. Second, we assume that it is hard to label problems or, equivalently, that it is hard to know who knows what. This implies that agents first try to solve the problem themselves and ask only if they do not know the answer. A worker with knowledge z asks with probability $1 - F(z)$.

We assume here that organizations can only have 1 or 2 layers, so that agents can choose to work on their own or work with other agents. Teams are formed by managers, who are specialized in problem solving, and production workers, who specialize in production and know only the routine tasks. Hence, a team of $n + 1$ agents, where n of them are production workers with knowledge z_p , has n units of production time available. The production of such a team is then simply $F(z_m)n$, where z_m is the knowledge of its manager. This is subject to the time constraint of the manager: the manager spends a fraction of time h communicating with each worker in the team whenever she cannot perform a task, which happens with probability $1 - F(z_p)$. The time constraint of managers is then given by: $hn(1 - F(z_p)) = 1$. Thus the span of the manager is limited by the knowledge of the production workers; if production workers are more knowledgeable, they will require help less often, and managers will be able to supervise larger teams. The rents of a manager are then given by $R(z_m, w(z_p)) = F(z_m)n - w(z_p)n$, where $w(z_p)$ is the wage of a worker with knowledge z_p .

The technology described above builds on the technology used in Lucas (1978) and Rosen (1982). Our focus on knowledge implies that, given the ability of the manager, teams with more knowledgeable workers are larger, since workers can solve more problems by themselves. In contrast, Lucas (1978) assumes that agents differ in managerial ability but are identical as workers. Rosen (1982) does have heterogenous workers, but only the efficiency units of skill matter for production. This assumes

away an important part of the matching problem between workers and managers that is central in this paper.

II SORTING AND ORGANIZATION

Solving for an equilibrium in this economy is a continuous assignment problem (see Sattinger (1993)) with two twists relative to standard assignment problems. First, who is assigned to whom is not a given, but an equilibrium outcome. In standard assignment models this identity is assumed. In contrast, here we are “marrying” a mass of workers with a mass of managers— *where those roles and masses are not given by assumption*. Second, agents can decide not to be matched and produce on their own.

To solve the assignment problem, note first that optimality requires positive sorting, that is, workers with more knowledge must be assigned to managers with more knowledge. The reason is that there is a complementarity between knowledge of workers and managers through the time constraint: a more knowledgeable manager will spread his higher knowledge over a larger number of workers, and that requires workers to be more knowledgeable so that they do not ask an excessive number of questions.¹

To characterize the equilibrium in this economy we need to describe three objects: the allocation of agents to occupations – production workers, specialized problem solvers, and self-employed; second, the team composition – i.e., the matching between workers and managers, and spans of control; and third, the earnings function. All these objects form an equilibrium, where earnings are such that agents do not want to switch either teams or occupations.

Under suitable conditions that we describe below, one can prove that an equilibrium will be characterized by a pair of thresholds (z_1, z_2) , such that all agents with knowledge $z < z_1$ become production workers, all agents with knowledge above z_2 become managers, and those in between are self-employed.

¹The result can be proven using the first and second order conditions (see Garicano and Rossi-Hansberg (2003)).

Suppose that a mass n of workers with knowledge z_p and a mass 1 of managers with knowledge z_m are matched together in a team. For this to be an equilibrium it must be the case that the assignment maximizes managerial rents, that is, that the manager would not be better off matching with either less knowledgeable or more knowledgeable workers. Manager's rents are given by

$$R(z_m, w) = \max_{z_p} n(F(z_m) - w(z_p)) = \max_{z_p} \frac{F(z_m) - w(z_p)}{[1 - F(z_p)] h},$$

where the second equality results from substituting in the time constraint of the manager. Thus a necessary condition for the assignment to be an equilibrium is that worker's wages be equal to the marginal value of worker's knowledge, that is,

$$w'(z_p) = \frac{F'(z_p)(F(z_m) - w(z_p))}{[1 - F(z_p)]} \text{ for all } z_p \leq z_1. \quad (1)$$

In equilibrium the market of production workers clears. Given wages and earnings, the supply and demand of production workers equalize, namely,

$$h \int_0^{z_p} [1 - F(t)] g(t) dt = G(m(z_p)) - G(m(0)) \text{ for all } z_p \leq z_1, \quad (2)$$

where $m(z_p)$ denotes the knowledge of the manager assigned to workers with knowledge z_p . Since (2) holds for all $z_p \leq z_1$, we can derive with respect to z_p , to obtain

$$m'(z_p) = h \frac{[1 - F(z_p)] g(z_p)}{g(m(z_p))}, \quad (3)$$

which, together with $m(0) = z_2$ and $m(z_1) = \bar{z}$, determines the equilibrium assignment function $m(z)$.

Finally, the occupational choice of agents must be optimal. Given equilibrium assignment and wage functions we can determine the rents of a manager with skill z_m , $R^*(z_m) = R(z_m, w(m^{-1}(z_m)))$. A worker can always choose to become self-employed and get $F(z)$. Thus, equilibrium earnings are given by $U(z) = \max\{F(z), R^*(z), w(z)\}$. This implies that the marginal production worker (the most knowledgeable one) must be indifferent between being a production worker or being self-employed, $w(z_1) = F(z_1)$, and the marginal manager (the least knowledgeable one) must be indifferent between being a manager and being self-employed, $R^*(z_2) = F(z_2)$. These are the final

and initial conditions needed to solve for the earnings function. An equilibrium of this type exists under some parameter restrictions.²

III COMMUNICATION COSTS, ORGANIZATION, AND INEQUALITY

The advantage of introducing a complete assignment model with a continuum of agents is that it allows us to talk about the effect of communication technology on within worker and manager class wage inequality, as well as on the type of teams formed in equilibrium. Throughout this section we assume that the distribution of knowledge in the population is uniform, $G(z) = z/\bar{z}$. When the distribution of problems is also uniform, so that all problems are equally frequent, we can obtain a closed form solution for the two differential equations in the above system and, as a result, fully characterize the changes in the distribution of firm sizes and in the distribution of earnings that follow from a change in communication technology, as the following proposition shows. We measure wage inequality as the ratio between the highest and lowest paid agent in a particular class.

Proposition 1 *Let $F(z) = z$, $\bar{z} = 1$, and let h be such that an equilibrium with two layers exist, then a decrease in communication costs (h) implies (1) a decrease in within worker class wage inequality, (2) an increase in within manager class wage inequality, (3) a decrease in the share of self-employed agents and (4) an increase in the share of workers and managers in the population.*

Proof. Due to space constraints we only present a sketch of the proof of these results. First, when $f(z)$ is uniform, team size is simply given by $n = 1/(h(1 - z_p))$. Substituting this in differential equation (3), together with the initial condition that $m(0) = z_2$, we obtain a quadratic expression for the knowledge of the manager, that is matched with a given production worker, as a function of the cut-off level z_2 . Replacing this in differential equation (1), together with the initial condition $w(z_1) = z_1$, we also obtain a quadratic differential equation for the equilibrium wages, $w(z)$, as a

²We need $z^F < \bar{z}$, where $h \int_0^{z^F} [1 - F(t)] g(t) dt = G(\bar{z}) - G(z^F)$.

function of both thresholds z_1 and z_2 . Solving $m(z_1) = 1$ in z_1 and substituting z_1 in $R(z_2) = z_2$, we obtain a single quadratic equation in z_2 from which we obtain closed form solutions for all the magnitudes in the model. Deriving with respect to h then yields the results. ■

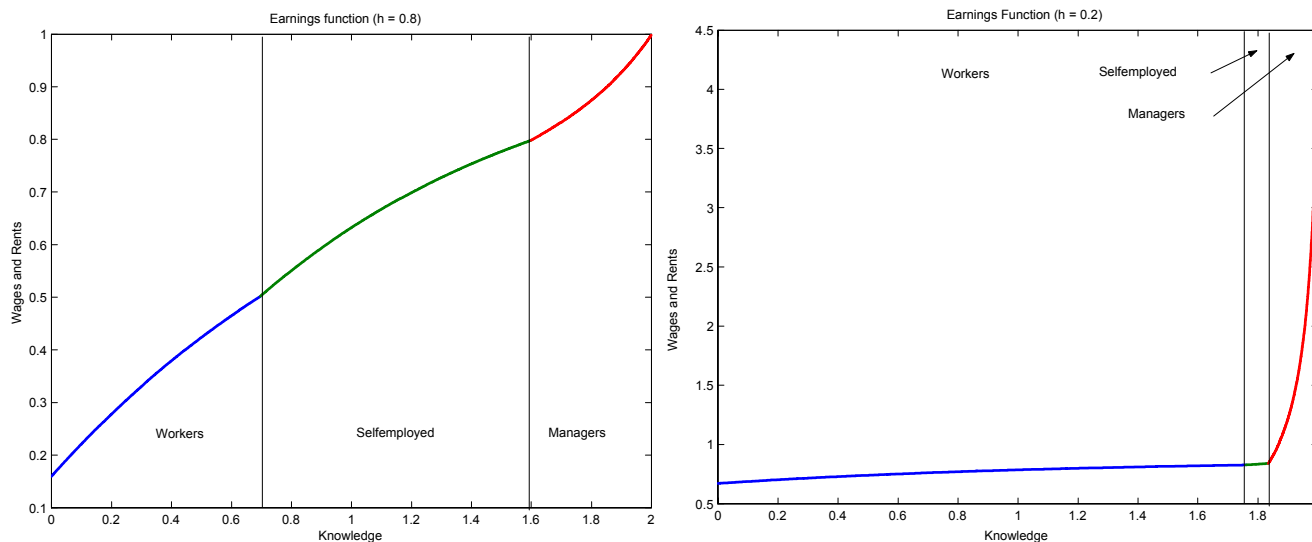
As we mentioned above, for an equilibrium with two layers to exist we need to guarantee that agents would not like to form teams with more than two layers. This restriction becomes binding for low values of h , where managers can leverage an increase in knowledge via a larger team. For $\bar{z} = 1$, in order to guarantee that an equilibrium with at most two layers exists, we need $h \geq 0.75$.

The effect of a decrease in communication costs on overall wage inequality is the result of combining the decrease in within worker class and the increase in within manager class wage inequality. Which of these two dominates depends heavily on the particular parameters and measure used. With the conditions imposed in the proposition above, we can show that overall wage inequality decreases. The proposition also shows that the number of organizations increases: more agents join teams since the technology to produce in teams improved.

All the results discussed above hold for a variety of distributions of tasks, although they are hard to prove analytically with more generality. We now illustrate the similar results with numerical exercises for an exponential distribution of tasks and a higher maximum amount of knowledge, $\bar{z} = 2$. In this case, higher z problems are less frequent. We first study the impact of a reduction in communication costs from $h = 0.8$ to $h = 0.2$. The results are presented in Figures 1 and 2.

The figures show the allocation of knowledge to different occupations and the associated earnings function. As we decrease h , the number of self-employed agents decreases. Lower costs of communicating knowledge imply that producing in teams is more efficient since managers can have larger spans of control and therefore leverage more their knowledge. Notice that, in contrast with the previous proposition, the share of managers in the economy decreases. The reason is that, with an exponential $F(z)$, high z problems are less frequent and therefore as h decreases less managers are needed. Within worker class wage inequality declines: the ratio of the highest paid worker to the lowest paid worker drops from 3.13 to 1.23. Wage inequality among

managers increases substantially: the ratio of the highest to the lowest paid manager goes from 1.25 to 5.04. Overall wage inequality, as measured by the ratio from the highest to the lowest paid worker, goes from 6.25 to 6.3: a modest increase.

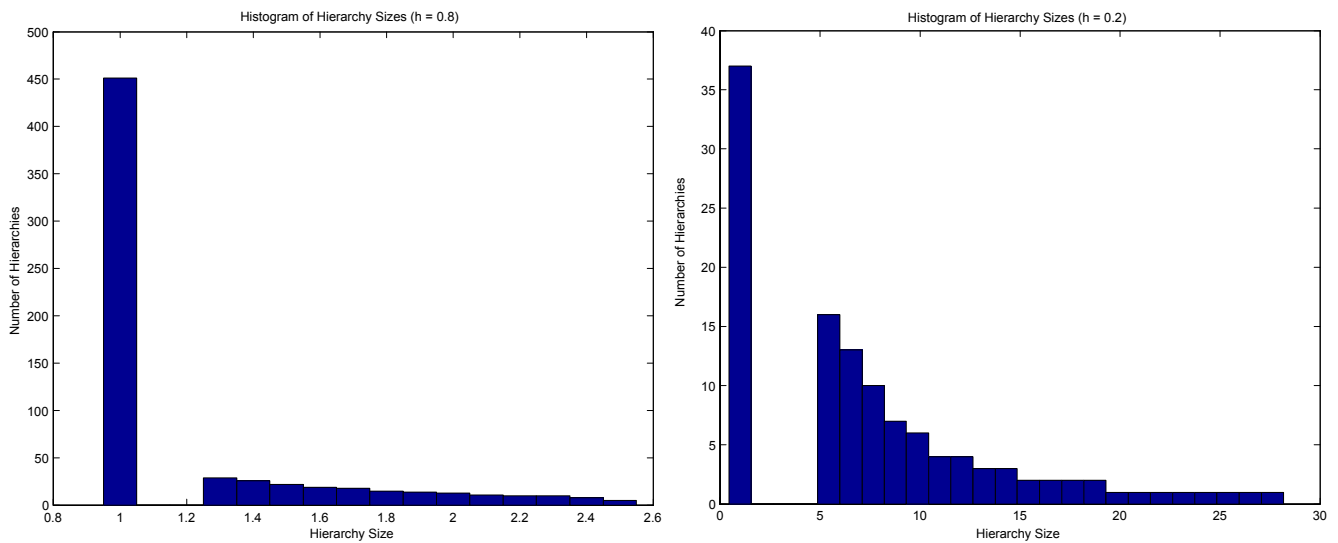


The intuition behind these results is straightforward. Inequality among workers decreases with h since spans of control increase and so workers with very different endowments of knowledge work for managers that have a similar (but not equal) amount of knowledge. This reduces the gain for workers of having more knowledge and therefore the level of wage inequality between them. The increase in spans, on the other hand, increases inequality among managers since extra knowledge can be leveraged more via a larger team. Overall wage inequality is the result of these two effects.

The exercises presented illustrate also the endogenous nature of organizations in the model. Organizations are not assumed but formed, because some of the agents in the economy benefit from producing in teams. This in turn determines equilibrium earnings. Endogenous organization, together with the heterogeneity in team formation via equilibrium assignments, yields a non-degenerate size distribution of hierarchies in the economy. In order to determine the number of hierarchies of each size we need to take a stand on the total mass of agents in the economy. In all the

figures presented we use a population size of 1000. Figures 3 and 4 illustrate the distribution of hierarchy sizes for $h = 0.8$ and 0.2 .

All self-employed agents constitute hierarchies with one layer. With high communication costs, all other hierarchies are rather small given that managers spend 80% of their time if they handle the problems of an agent with zero knowledge. The distribution of hierarchy sizes is skewed and declining. The gap between hierarchies with 1 employee and hierarchies with more than 1 employee is the result of having a discrete number of layers. With higher communication costs, the size distribution of hierarchies shows much more variation and declines at a decreasing rate, consistent with a power distribution. Hierarchies are much larger, the largest hierarchy has 28 employees. Naturally, maximum team size increases with \bar{z} .



IV CONCLUSION

We presented a model in which agents with heterogenous levels of knowledge form hierarchical teams that may differ in the knowledge of their employees and managers, in spans of control, and in the number of layers (self-employed agents or two layer hierarchies). We showed that reductions in communication costs lead to more inequality between managers and less inequality between workers. This is associated with an organizational change characterized by more and larger hierarchies, and a

decrease in the share of self-employed agents.

We have abstracted from the problem of knowledge acquisition. In fact, the distribution of knowledge is the result of agent's investments in learning, which are affected by, and determine, the organizational structure and earnings schedule. In Garicano and Rossi-Hansberg (2003) we incorporate this important dimension into the analysis.

The general point of our theory is that understanding the reasons why organizations form, and the resulting characteristics of these organizations, is important to understand the way in which technological change will affect aggregate economic variables and the way agents organize production in the economy. Our analysis takes a step in this direction by studying the effect of communication technology on wage inequality and organizational structure.

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